Hamilton's equations in Cartesian Coordinates:

For generalized coordinates $q_1, q_2, \ldots q_k, \ldots q_n$ and generalized momenta $p_1, p_2, \ldots p_k, \ldots p_n$ respectively, the Hamilton's equations of motion are given by

$$\dot{q}_k = \frac{\partial H}{\partial p_k} \quad \text{—— (1)}$$

$$-\dot{p}_k = \frac{\partial H}{\partial q_k} \quad \text{—— (2)}$$

We obtain these equations in Cartesian coordinate.

The kinetic energy in Cartesian coordinate is given by $T = \frac{1}{2} m (\dot{x}^2 + \dot{y}^2 + \dot{z}^2)$

Let the potential energy is $V = V(x, y, z)$.

The Lagrangian L is given by, $L = T - V$;

$$L = \frac{1}{2} m (\dot{x}^2 + \dot{y}^2 + \dot{z}^2) - V(x, y, z) \quad \text{—— (3)}$$

We know that generalized momentum can be defined by $p_k = \frac{\partial L}{\partial \dot{q}_k}$. Thus, from equation we obtain

$$p_x = \frac{\partial L}{\partial \dot{x}} = m\dot{x}$$

or $$\dot{x} = \frac{p_x}{m} \quad \text{—— (4)}$$

Similarly, $$\dot{y} = \frac{p_y}{m} \quad \text{—— (5)}$$

$$\dot{z} = \frac{p_z}{m} \quad \text{—— (6)}$$

Hamiltonian H is given by

$$H = \sum_{k=x,y,z} p_k \dot{q}_k - L$$

or

$$H = p_x \dot{x} + p_y \dot{y} + p_z \dot{z} - \frac{1}{2} m (\dot{x}^2 + \dot{y}^2 + \dot{z}^2) + V(x,y,z)$$

From eqs. (4), (5) and (6)

$$H = p_x\left(\frac{p_x}{m}\right) + p_y\left(\frac{p_y}{m}\right) + p_z\left(\frac{p_z}{m}\right) - \frac{1}{2} m \left(\frac{p_x^2}{m^2} + \frac{p_y^2}{m^2} + \frac{p_z^2}{m^2}\right) + V(x,y,z)$$

or $$H = \frac{p_x^2}{m} + \frac{p_y^2}{m} + \frac{p_z^2}{m} - \frac{1}{2m}(p_x^2 + p_y^2 + p_z^2) + V(x,y,z)$$

$$H = \frac{1}{2m}(p_x^2 + p_y^2 + p_z^2) + V(x,y,z) \quad \text{—— (7)}$$

Now Hamilton's equations in cartesian coordinates are given by

$$\dot{x} = \frac{\partial H}{\partial p_y},$$

$$\dot{p}_x = -\frac{\partial H}{\partial x}$$

or $$\dot{x} = \frac{p_x}{m}, \quad \dot{p}_x = -\frac{\partial V}{\partial x}$$

Similarly $$\left.\begin{aligned} \dot{y} &= \frac{p_y}{m}, \quad \dot{p}_y = -\frac{\partial V}{\partial y} \\ \dot{z} &= \frac{p_z}{m}, \quad \dot{p}_z = -\frac{\partial V}{\partial z} \end{aligned}\right\} \text{—— (8)}$$

The above equations (eq. 8) can be written as

$$\boxed{m\ddot{x} = -\frac{\partial V}{\partial x}, \quad m\ddot{y} = -\frac{\partial V}{\partial y}, \quad m\ddot{z} = -\frac{\partial V}{\partial z}}$$